



• Naïve Bayes and Classifying

COSI 114 – Computational Linguistics
James Pustejovsky

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Brandeis University

Chapter 6, Jurafsky and Martin (2015) SLP

Is this spam?

Subject: Important notice!

From: Stanford University <newsforum@stanford.edu>

Date: October 28, 2011 12:34:16 PM PDT

To: undisclosed-recipients;;

Greats News!

You can now access the latest news by using the link below to login to Stanford University News Forum.

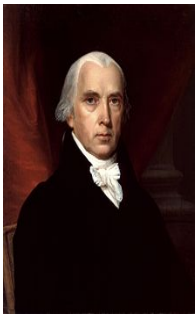
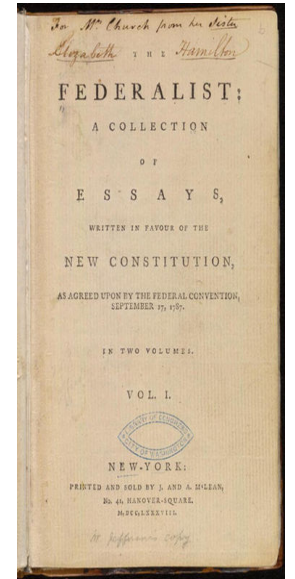
<http://www.123contactform.com/contact-form-StanfordNew1-236335.html>

Click on the above link to login for more information about this new exciting forum. You can also copy the above link to your browser bar and login for more information about the new services.

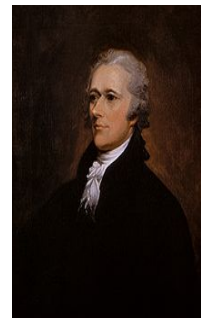
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Who wrote which Federalist papers?

- 1787-8: anonymous essays try to convince New York to ratify U.S Constitution: Jay, Madison, Hamilton.
- Authorship of 12 of the letters in dispute
- 1963: solved by Mosteller and Wallace using Bayesian methods



James Madison



Alexander Hamilton

Male or female author?

1. By 1925 present-day Vietnam was divided into three parts under French colonial rule. The southern region embracing Saigon and the Mekong delta was the colony of Cochinchina; the central area with its imperial capital at Hue was the protectorate of Annam...
2. Clara never failed to be astonished by the extraordinary felicity of her own name. She found it hard to trust herself to the mercy of fate, which had managed over the years to convert her greatest shame into one of her greatest assets...

Positive or negative movie review?



- unbelievably disappointing



- Full of zany characters and richly applied satire, and some great plot twists



- this is the greatest screwball comedy ever filmed



- It was pathetic. The worst part about it was the boxing scenes.

What is the subject of this article?

MEDLINE Article



MeSH Subject Category Hierarchy

- Antagonists and Inhibitors
- Blood Supply
- Chemistry
- Drug Therapy
- Embryology
- Epidemiology
- ...

?



Text Classification

- Assigning subject categories, topics, or genres
- Spam detection
- Authorship identification
- Age/gender identification
- Language Identification
- Sentiment analysis
- ...

Text Classification: definition

- *Input:*
 - a document d
 - a fixed set of classes $C = \{c_1, c_2, \dots, c_J\}$
- *Output:* a predicted class $c \in C$



Classification Methods: Hand-coded rules

- Rules based on combinations of words or other features
 - spam: black-list-address OR (“dollars” AND “have been selected”)
- Accuracy can be high
 - If rules carefully refined by expert
- But building and maintaining these rules is expensive

Classification Methods:

Supervised Machine Learning

- *Input:*
 - a document d
 - a fixed set of classes $C = \{c_1, c_2, \dots, c_J\}$
 - A training set of m hand-labeled documents $(d_1, c_1), \dots, (d_m, c_m)$
- *Output:*
 - a learned classifier $\gamma: d \rightarrow c$



Classification Methods: Supervised Machine Learning

- Any kind of classifier
 - Naïve Bayes
 - Logistic regression
 - Support-vector machines
 - k-Nearest Neighbors
 - ...



Text Classification and Naïve Bayes

The Task of Text Classification



Text Classification and Naïve Bayes

Naïve Bayes (I)

Naïve Bayes Intuition

- Simple (“naïve”) classification method based on Bayes rule
- Relies on very simple representation of document
 - Bag of words

The bag of words representation

Y (

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet.

) = C





The bag of words representation

Y (I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet.) = C



The bag of words representation: using a subset of words

$$Y \left(\begin{array}{l} \text{x love xxxxxxxxxxxxxxxx sweet} \\ \text{xxxxxxxx satirical xxxxxxxxxxxx} \\ \text{xxxxxxxxxxxx great xxxxxxxx} \\ \text{xxxxxxxxxxxxxxxxxxxxxxxx fun xxxx} \\ \text{xxxxxxxxxxxxxxxx whimsical xxxx} \\ \text{romantic xxxx laughing} \\ \text{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx} \\ \text{xxxxxxxxxxxxxxxx recommend xxxxx} \\ \text{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx} \\ \text{xx several xxxxxxxxxxxxxxxxxxxx} \\ \text{xxxxx happy xxxxxxxx again} \\ \text{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx} \\ \text{xxxxxxxxxxxxxxxxxxxxxxxx} \end{array} \right) = C$$


The bag of words representation

$Y(\text{Table}) = C$

great	2
love	2
recommend	1
laugh	1
happy	1
...	...

thumbs up
thumbs down

Bag of words for document classification

?

Test
document

parser
language
label
translation
...

Machine
Learning

learning
training
algorithm
shrinkage
network...

NLP

parser
tag
training
translation
language...

Garbage
Collection

garbage
collection
memory
optimization
region...

Planning

planning
temporal
reasoning
plan
language...

GUI

...



Text Classification and Naïve Bayes

Naïve Bayes (I)



Text Classification and Naïve Bayes

Formalizing the Naïve Bayes Classifier

Bayes' Rule Applied to Documents and Classes

- For a document d and a class c

$$P(c | d) = \frac{P(d | c)P(c)}{P(d)}$$

Naïve Bayes Classifier (I)

$$c_{MAP} = \operatorname{argmax}_{c \in C} P(c | d)$$

MAP is “maximum a posteriori” = most likely class

$$= \operatorname{argmax}_{c \in C} \frac{P(d | c)P(c)}{P(d)}$$

Bayes Rule

$$= \operatorname{argmax}_{c \in C} P(d | c)P(c)$$

Dropping the denominator

Naïve Bayes Classifier (II)

$$c_{MAP} = \operatorname{argmax}_{c \in C} P(d | c)P(c)$$

$$= \operatorname{argmax}_{c \in C} P(x_1, x_2, \dots, x_n | c)P(c)$$

Document d
represented as
features $x_1 \dots x_n$

Naïve Bayes Classifier (IV)

$$c_{MAP} = \operatorname{argmax}_{c \in C} P(x_1, x_2, \dots, x_n | c) P(c)$$

$O(|X|^n \cdot |C|)$ parameters

How often does this class occur?

Could only be estimated if a very, very large number of training examples was available.

We can just count the relative frequencies in a corpus

Multinomial Naïve Bayes

Independence Assumptions

$$P(x_1, x_2, \dots, x_n | c)$$

- **Bag of Words assumption:** Assume position doesn't matter
- **Conditional Independence:** Assume the feature probabilities $P(x_i | c_j)$ are independent given the class c .

$$P(x_1, \dots, x_n | c) = P(x_1 | c) \cdot P(x_2 | c) \cdot P(x_3 | c) \cdot \dots \cdot P(x_n | c)$$

Multinomial Naïve Bayes Classifier

$$c_{MAP} = \operatorname{argmax}_{c \in C} P(x_1, x_2, \dots, x_n | c) P(c)$$

$$c_{NB} = \operatorname{argmax}_{c \in C} P(c_j) \prod_{x \in X} P(x | c)$$

Applying Multinomial Naive Bayes Classifiers to Text Classification

positions \leftarrow all word positions in test document

$$c_{NB} = \operatorname{argmax}_{c_j \in C} P(c_j) \prod_{i \in \text{positions}} P(x_i | c_j)$$



Text Classification and Naïve Bayes

Formalizing the Naïve Bayes Classifier



Text Classification and Naïve Bayes

Naïve Bayes: Learning

Learning the Multinomial Naïve Bayes Model

- First attempt: maximum likelihood estimates
 - simply use the frequencies in the data

$$\hat{P}(c_j) = \frac{\text{doccount}(C = c_j)}{N_{doc}}$$

$$\hat{P}(w_i | c_j) = \frac{\text{count}(w_i, c_j)}{\sum_{w \in V} \text{count}(w, c_j)}$$

Parameter estimation

$$\hat{P}(w_i | c_j) = \frac{\text{count}(w_i, c_j)}{\sum_{w \in V} \text{count}(w, c_j)}$$

fraction of times word w_i appears
among all words in documents of topic c_j

- Create mega-document for topic j by concatenating all docs in this topic
 - Use frequency of w in mega-document

Problem with Maximum Likelihood

- What if we have seen no training documents with the word *fantastic* and classified in the topic **positive** (*thumbs-up*)?

$$\hat{P}(\text{"fantastic"} \mid \text{positive}) = \frac{\text{count}(\text{"fantastic"}, \text{positive})}{\sum_{w \in V} \text{count}(w, \text{positive})} = 0$$

- Zero probabilities cannot be conditioned away, no matter the other evidence!

$$c_{MAP} = \operatorname{argmax}_c \hat{P}(c) \prod_i \hat{P}(x_i \mid c)$$

Laplace (add-1) smoothing for Naïve Bayes

$$\begin{aligned}\hat{P}(w_i | c) &= \frac{\text{count}(w_i, c) + 1}{\sum_{w \in V} (\text{count}(w, c) + 1)} \\ &= \frac{\text{count}(w_i, c) + 1}{\left(\sum_{w \in V} \text{count}(w, c) \right) + |V|}\end{aligned}$$

Multinomial Naïve Bayes: Learning

- From training corpus, extract *Vocabulary*
- Calculate $P(c_j)$ terms
 - For each c_j in C do
 - $docs_j \leftarrow$ all docs with class $= c_j$
- Calculate $P(w_k | c_j)$ terms
 - $Text_j \leftarrow$ single doc containing all $docs_j$
 - For each word w_k in *Vocabulary*
 - $n_k \leftarrow$ # of occurrences of w_k in $Text_j$

$$P(c_j) \leftarrow \frac{|docs_j|}{|\text{total \# documents}|}$$

$$P(w_k | c_j) \leftarrow \frac{n_k + \alpha}{n + \alpha |Vocabulary|}$$



Text Classification and Naïve Bayes

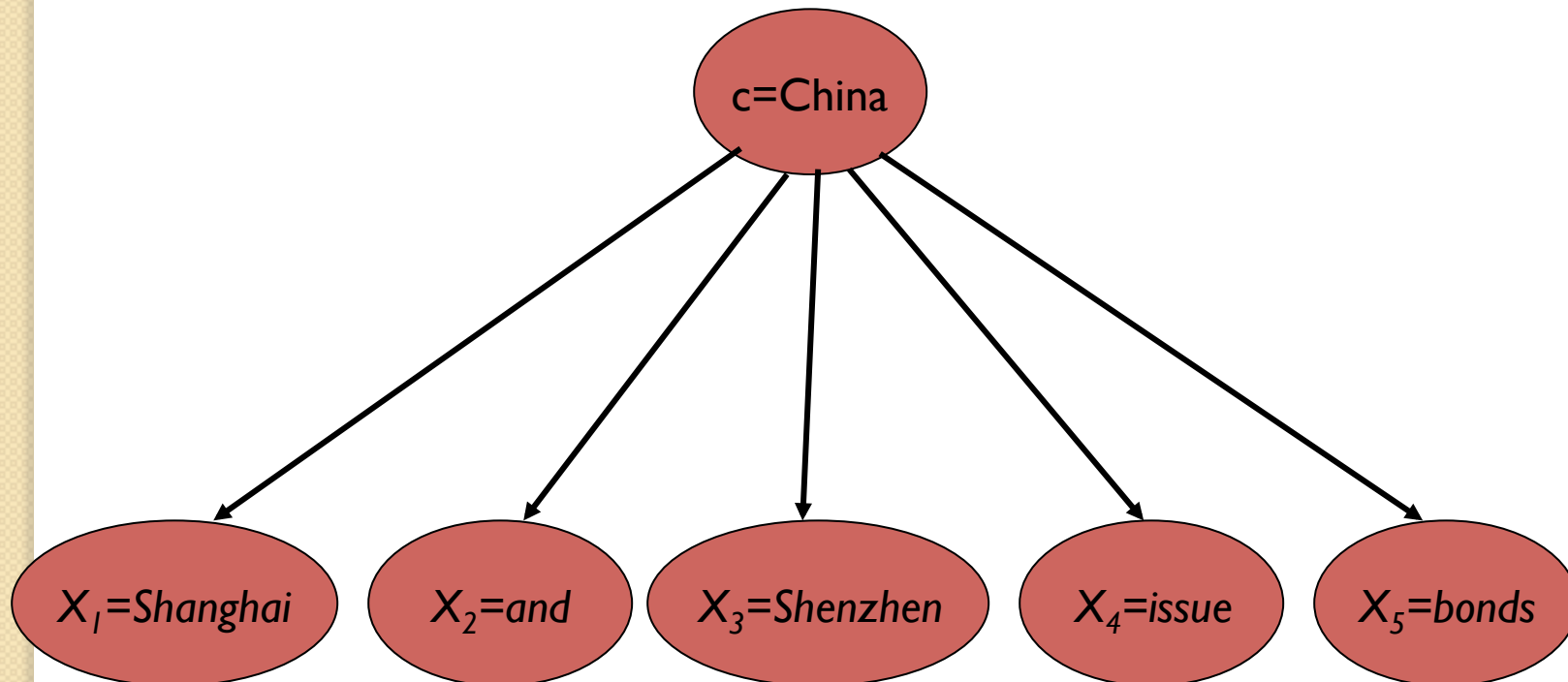
Naïve Bayes: Learning



Text Classification and Naïve Bayes

Naïve Bayes: Relationship to Language
Modeling

Generative Model for Multinomial Naïve Bayes



Naïve Bayes and Language Modeling

- Naïve bayes classifiers can use any sort of feature
 - URL, email address, dictionaries, network features
- But if, as in the previous slides
 - We use **only** word features
 - we use **all** of the words in the text (not a subset)
- Then
 - Naïve bayes has an important similarity to language modeling.

Each class = a unigram language model

- Assigning each word: $P(\text{word} \mid c)$
- Assigning each sentence: $P(s \mid c) = \prod P(\text{word} \mid c)$

Class *pos*

0.1 I	I	love	this	fun	film
0.1 love	_____	_____	_____	_____	_____
0.01this	0.1	0.1	.05	0.01	0.1
0.05fun					
0.1 film					
...					

$$P(s \mid \text{pos}) = 0.0000005$$

Naïve Bayes as a Language Model

- Which class assigns the higher probability to s?

Model pos

0.1	I
0.1	love
0.01	this
0.05	fun
0.1	film

Model neg

0.2	I
0.001	love
0.01	this
0.005	fun
0.1	film

I	love	this	fun	film
0.1	0.1	0.01	0.05	0.1
0.2	0.001	0.01	0.005	0.1

$$P(s|\text{pos}) > P(s|\text{neg})$$



Text Classification and Naïve Bayes

Naïve Bayes: Relationship to Language
Modeling



Text Classification and Naïve Bayes

Multinomial Naïve Bayes: A Worked Example

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(w|c) = \frac{\text{count}(w,c)+1}{\text{count}(c)+|V|}$$

Priors:

$$P(c) = \frac{3}{4}$$

$$P(j) = \frac{1}{4}$$

	D oc	Words	Clas s
Trainin g	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

Choosing a class:

$$P(c|d5) \propto \frac{3}{4} * \left(\frac{3}{7}\right)^3 * \frac{1}{14} * \frac{1}{14} \approx 0.0003$$

Conditional Probabilities:

$$P(\text{Chinese}|c) = (5+1) / (8+6) = 6/14 = 3/7$$

$$P(\text{Tokyo}|c) = (0+1) / (8+6) = 1/14$$

$$P(\text{Japan}|c) = (0+1) / (8+6) = 1/14$$

$$P(\text{Chinese}|j) = (0+1) / (8+6) = 1/14$$

$$P(\text{Tokyo}|j) = (1+1) / (3+6) = 2/9$$

$$P(\text{Japan}|j) = (1+1) / (3+6) = 2/9$$

$$(1+1) / (3+6) = 2/9$$

$$P(j|d5)$$

$$\propto \frac{1}{4} * \left(\frac{2}{9}\right)^3 * \frac{2}{9} * \frac{2}{9} \approx 0.0001$$

Naïve Bayes in Spam Filtering

- SpamAssassin Features:

- Mentions Generic Viagra
- Online Pharmacy
- Mentions millions of (dollar) ((dollar) NN,NNN,NNN.NN)
- Phrase: impress ... girl
- From: starts with many numbers
- Subject is all capitals
- HTML has a low ratio of text to image area
- One hundred percent guaranteed
- Claims you can be removed from the list
- 'Prestigious Non-Accredited Universities'
- http://spamassassin.apache.org/tests_3_3_x.html

Summary: Naive Bayes is Not So Naive

- Very Fast, low storage requirements
- Robust to Irrelevant Features

Irrelevant Features cancel each other without affecting results

- Very good in domains with many equally important features

Decision Trees suffer from *fragmentation* in such cases – especially if little data

- Optimal if the independence assumptions hold: If assumed independence is correct, then it is the Bayes Optimal Classifier for problem
- A good dependable baseline for text classification
 - **But we will see other classifiers that give better accuracy**



Text Classification and Naïve Bayes

Multinomial Naïve Bayes: A Worked Example



Text Classification and Naïve Bayes

Precision, Recall, and the F measure

The 2-by-2 contingency table

	correct	not correct
selected	tp	fp
not selected	fn	tn

Precision and recall

- **Precision:** % of selected items that are correct

Recall: % of correct items that are selected

	correct	not correct
selected	tp	fp
not selected	fn	tn

A combined measure: F

- A combined measure that assesses the P/R tradeoff is F measure (weighted harmonic mean):

$$F = \frac{1}{\alpha \frac{1}{P} + (1-\alpha) \frac{1}{R}} = \frac{(\beta^2 + 1)PR}{\beta^2 P + R}$$

- The harmonic mean is a very conservative average;
- People usually use balanced FI measure
 - i.e., with $\beta = 1$ (that is, $\alpha = 1/2$):
 - $F = 2PR/(P+R)$

More Than Two Classes: Sets of binary classifiers

- Dealing with **any-of** or **multivalue** classification
 - A document can belong to 0, 1, or >1 classes.
- For each class $c \in C$
 - Build a classifier γ_c to distinguish c from all other classes $c' \in C$
- Given test doc d ,
 - Evaluate it for membership in each class using each γ_c
 - d belongs to **any** class for which γ_c returns true

More Than Two Classes: Sets of binary classifiers

- One-of or multinomial classification
 - Classes are mutually exclusive: each document in exactly one class
- For each class $c \in C$
 - Build a classifier γ_c to distinguish c from all other classes $c' \in C$
- Given test doc d ,
 - Evaluate it for membership in each class using each γ_c
 - d belongs to the one class with maximum score

Evaluation: Classic Reuters-21578 Data Set

- Most (over)used data set, 21,578 docs (each 90 types, 200 tokens)
- 9603 training, 3299 test articles (ModApte/Lewis split)
- 118 categories
 - An article can be in more than one category
 - Learn 118 binary category distinctions
- Average document (with at least one category) has 1.24 classes
- Only about 10 out of 118 categories are large

Common categories
(#train, #test)

- Earn (2877, 1087)
- Acquisitions (1650, 179)
- Money-fx (538, 179)
- Grain (433, 149)
- Crude (389, 189)

- Trade (369,119)
- Interest (347, 131)
- Ship (197, 89)
- Wheat (212, 71)
- Corn (182, 56)

Reuters Text Categorization data set (Reuters-21578) document

<REUTERS TOPICS="YES" LEWISSPLIT="TRAIN" CGISPLIT="TRAINING-SET" OLDID="12981" NEWID="798">

<DATE> 2-MAR-1987 16:51:43.42</DATE>

<TOPICS><D>livestock</D><D>hog</D></TOPICS>

<TITLE>AMERICAN PORK CONGRESS KICKS OFF TOMORROW</TITLE>

<DATELINE> CHICAGO, March 2 - </DATELINE><BODY>The American Pork Congress kicks off tomorrow, March 3, in Indianapolis with 160 of the nations pork producers from 44 member states determining industry positions on a number of issues, according to the National Pork Producers Council, NPPC.

Delegates to the three day Congress will be considering 26 resolutions concerning various issues, including the future direction of farm policy and the tax law as it applies to the agriculture sector. The delegates will also debate whether to endorse concepts of a national PRV (pseudorabies virus) control and eradication program, the NPPC said.

A large trade show, in conjunction with the congress, will feature the latest in technology in all areas of the industry, the NPPC added. Reuter

</BODY></TEXT></REUTERS>

Confusion matrix c

- For each pair of classes $\langle c_1, c_2 \rangle$ how many documents from c_1 were incorrectly assigned to c_2 ?
 - $c_{3,2}$: 90 wheat documents incorrectly assigned to poultry

Docs in test set	Assigned UK	Assigned poultry	Assigned wheat	Assigned coffee	Assigned interest	Assigned trade
True UK	95	1	13	0	1	0
True poultry	0	1	0	0	0	0
True wheat	10	90	0	1	0	0
True coffee	0	0	0	34	3	7
True interest	-	1	2	13	26	5
True trade	0	0	2	14	5	10

Per class evaluation measures

Recall:

Fraction of docs in class i classified correctly:

$$\frac{c_{ii}}{\sum_j c_{ij}}$$

Precision:

Fraction of docs assigned class i that are actually about class i :

$$\frac{c_{ii}}{\sum_j c_{ji}}$$

Accuracy: (1 - error rate)

Fraction of docs classified correctly:

$$\frac{\sum_i c_{ii}}{\sum_j \sum_i c_{ij}}$$

Micro- vs. Macro-Averaging

- If we have more than one class, how do we combine multiple performance measures into one quantity?
- **Macroaveraging:** Compute performance for each class, then average.
- **Microaveraging:** Collect decisions for all classes, compute contingency table, evaluate.

Micro- vs. Macro-Averaging: Example

Class 1

	Truth: yes	Truth: no
Classifier: yes	10	10
Classifier: no	10	970

Class 2

	Truth: yes	Truth : no
Classifier: yes	90	10
Classifier: no	10	890

Micro Ave. Table

	Truth: yes	Truth: no
Classifier: yes	100	20
Classifier: no	20	1860

- Macroaveraged precision: $(0.5 + 0.9)/2 = 0.7$
- Microaveraged precision: $100/120 = .83$
- Microaveraged score is dominated by score on common classes

Development Test Sets and Cross-validation

Training set

Development Test Set

Test Set

- Metric: P/R/F1 or Accuracy
- Unseen test set
 - avoid overfitting ('tuning to the test set')
 - more conservative estimate of performance
- Cross-validation over multiple splits
 - Handle sampling errors from different datasets
 - Pool results over each split
 - Compute pooled dev set performance

Training Set

Dev Test

Training Set

Dev Test

Dev Test

Training Set

Test Set

The Real World

- Gee, I'm building a text classifier for real, now!
- What should I do?

No training data?

Manually written rules

If (wheat or grain) and not (whole or bread) then
Categorize as grain

- Need careful crafting
 - Human tuning on development data
 - Time-consuming: 2 days per class

Very little data?

- Use Naïve Bayes
 - Naïve Bayes is a “high-bias” algorithm (Ng and Jordan 2002 NIPS)
- Get more labeled data
 - Find clever ways to get humans to label data for you
- Try semi-supervised training methods:
 - Bootstrapping, EM over unlabeled documents, ...

A reasonable amount of data?

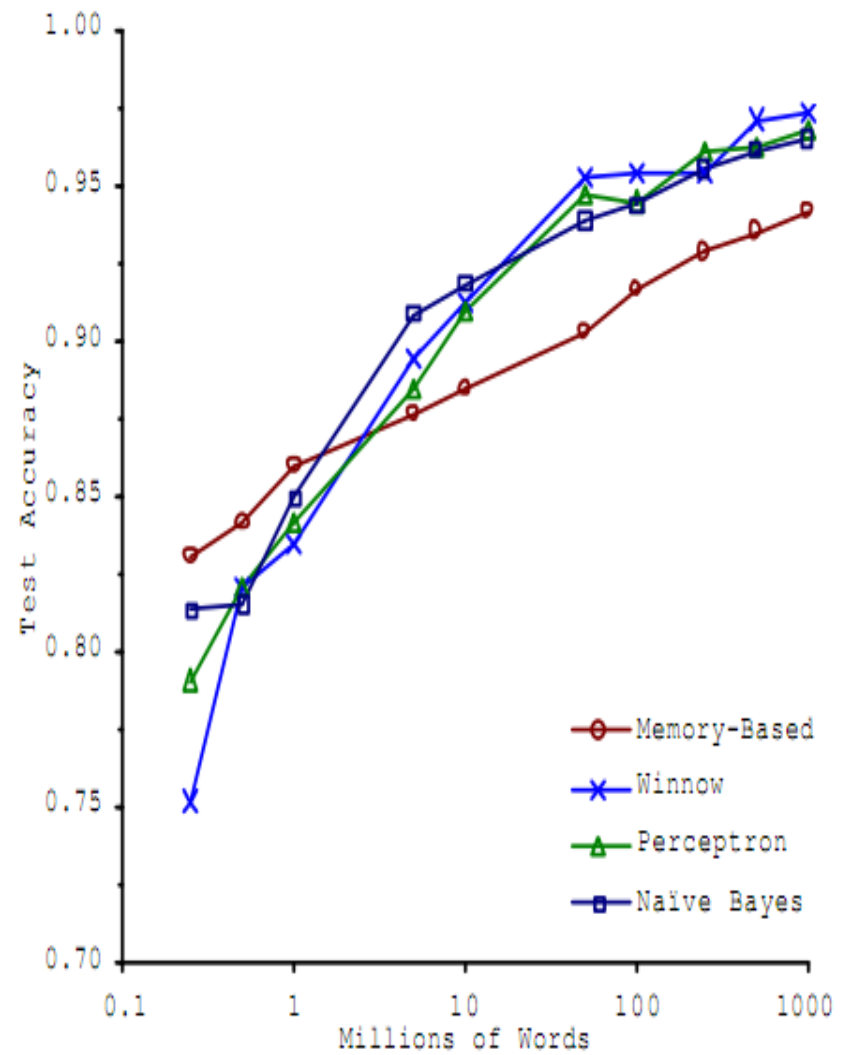
- Perfect for all the clever classifiers
 - SVM
 - Regularized Logistic Regression
- You can even use user-interpretable decision trees
 - Users like to hack
 - Management likes quick fixes

A huge amount of data?

- Can achieve high accuracy!
- At a cost:
 - SVMs (train time) or kNN (test time) can be too slow
 - Regularized logistic regression can be somewhat better
- So Naïve Bayes can come back into its own again!

Accuracy as a function of data size

- With enough data
 - Classifier may not matter





Real-world systems generally combine:

- Automatic classification
- Manual review of uncertain/difficult/"new" cases

Underflow Prevention: log space

- Multiplying lots of probabilities can result in floating-point underflow.
- Since $\log(xy) = \log(x) + \log(y)$
 - Better to sum logs of probabilities instead of multiplying probabilities.
- Class with highest un-normalized log probability score is still most probable.

$$c_{NB} = \operatorname{argmax}_{c_j \in C} \log P(c_j) + \sum_i \log P(x_i | c_j)$$

- Model is now just max of sum of weights

How to tweak performance

- Domain-specific features and weights: *very* important in real performance
- Sometimes need to collapse terms:
 - Part numbers, chemical formulas, ...
 - But stemming generally doesn't help
- Upweighting: Counting a word as if it occurred twice:
 - title words (Cohen & Singer 1996)
 - first sentence of each paragraph (Murata, 1999)
 - In sentences that contain title words (Ko *et al*, 2002)